is the curve of intersection of the paraboloid and the cylinder. This curve divides the given surface into two parts. The particle will be in equilibrium at any point of the lower and at no point of the upper.

Two excellent solutions of this problem were received from *F. P. Matz*, and one from *G. B. M. Zerr*.

PROBLEMS.

26. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

If an elastic sphere be electrified in such a manner that the initial internal pressure remains constant, determine an expression for the *ratio of the electrical densities* when the volume of the sphere has been increased to $(m+1)$ times its initial volume.

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

20. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton. Virginia.

Find two integral numbers, whose sum, difference, and difference of their squares shall be a square, cube, and fourth power.

I. Solution by J. H. DRUMMOND; H. C. WILKES; and M. A. GRUBER.

Let x and y=the two integral numbers. Any number to be a square, a cube, and a fourth power, must also be a twelfth power.

$$\text{Then } x+y=a^{12}$$
$$x-y=b^{12}$$
$$x^2-y^2=a^{12}b^{12}=(ab)^{12}.$$
$$\text{Whence } x=\tfrac{1}{2}(a^{12}+b^{12}), \text{ and } y=\tfrac{1}{2}(a^{12}-b^{12}).$$

In order that x and y be integral, a^{12} and b^{12} must be both odd or both even.

Put $a^{12}=5^{12}=244,140,625$ and $b^{12}=3^{12}=531441$. Then $x=122,336,033$ and $y=121,804,592$. Put $a^{12}=6^{12}=2,176,782,336$ and $b^{12}=2^{12}=4096$. Then $x=1,088,393,216$ and $y=1,088,389,120$.

The lowest values of x and y are found by putting

$$x+y=a^{12}=3^{12}=531441,$$
$$x-y=b^{12}=1^{12}=1.$$
$$\text{Whence } x=265721 \text{ and } y=265720.$$

Many answers can be obtained but the work will be tedious.

II. **Solution by the PROPOSER.**

Let, $x^m+\dfrac{m(m-1)}{2}x^{m-2}y^2+\ldots.+\dfrac{m(m-1)}{2}x^2y^{m-2}+y^m$, and $mx^{m-1}y+$

$\dfrac{m(m-1)(m-2)}{2.3}x^{m-3}y^3+\ldots.+\dfrac{m(m-1)(m-2)}{2.3}x^3y^{m-3}+mxy^{m-1}$ be the numbers.

Then $(x+y)^m$, $(x-y)^m$, $(x^2-y^2)^m$, is their sum, their difference, and the difference of their squares. m must be divisible by 2, 3, and 4; this is the case when $m=12$. Then the numbers are $x^{12}+66x^{10}y^2+\ldots+66x^2y^{10}+y^{12}$,and $12x^{11}y+220x^9y^3+\ldots.+220x^3y^9+12xy^{11}$ and $(x+y)^{12}$, $(x-y)^{12}$, $(x^2-y^2)^{12}$ is their sum, their difference, and the difference of their squares.

Let $x=2$, $y=1$, then the numbers are 265721, 265720. Their sum $=(3)^{12}=531441=(729)^2=(81)^3=(27)^4$. Their difference $=1=(1)^2=(1)^3=(1)^4$. Difference of their squares $=531441=(729)^2=(81)^3=(27)^4$.

Many other numbers can be found satisfying the conditions.

Also solved by *H. W. Draughon, and J. F. W. Scheffer.*

21. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.**

Find (1) nine positive *integral numbers* in arithmetical progression the sum of whose squares is a *square number*; and (2) find nine integral *square numbers* whose sum is a *square number*.

I. **Solution by H. W. DRAUGHON, Olio, Mississippi.**

(1). Let, $x+4y$, $x+3y$,$x+2y$, $x+y$, x,$x-y$,$x-2y$,$x-3y$, and $x-4y$ be the numbers. Then we are to make the sum of their squares, $9x^2+60y^2=\square$. Let us assume, $9x^2+60y^2=(3x+6m)^2=9x^2+36mx+36m^2$; then,

$x=\dfrac{60y^2-36m^2}{36m}=\dfrac{5y^2}{3m}-m$. In order that x may be integral put, $y=3pm\ldots(1)$.

Then, $x=15p^2m-m=(15p^2-1)m\ldots.(2)$. p and m can have any positive, integral values that will make $x>4y$.

Let us make, $p=1$ and $m=1$; then, $x=14$, $y=3$, and the numbers are, 2, 5, , 11, 14, 17, 20, 23, and 26. The sum of their squares is $(48)^2$.

Again, let, $p=2$, and $m=1$; then, $x=59$, $y=12$, and the numbers are, 11, 23, 35, 47, 59, 71, 83, 95, and 107. The sum of the squares of this set is $(183)^2$. An infinite number of sets can be thus obtained from (2).

(2). In the *Mathematical Messenger*, Vol. 7, No. 5. page 47, I find the following formula for n square numbers whose sum is a square:

$S+\frac{1}{4}(p-q)^2=\frac{1}{4}(p+q)^2$, in which $S=pq=$the sum of $n-1$ square numbers. Here, $n=9$. Let us assume, $S=(3)^2+(4)^2+(5)^2+(8)^2+(9)^2+(10)^2+(11)^2+(12)^2=560=10\times56$. Let us make $p=56$ and $q=10$; then, we have, $S+(23)^2=(33)^2$. Any other factors of 560 may be taken. When factors give fractional results we clear of fractions.

II. **Solution by R. J. ADCOCK, Larchland, Illinois.**